Chapter 1.1 Rational and Irrational Numbers

A **rational number** is a number that can be written as a ratio or the quotient of two integers \( a \) and \( b \) written \( a/b \) where \( b \neq 0 \).

Integers, fractions and mixed numbers, decimals both positive and negative and terminating and repeating are all rational numbers.

Multiplying a number by itself is called finding the square of a number. Perfect squares or square numbers are the squares of the natural numbers. Natural numbers are your counting numbers 1,2,3,4,5......

A **square root of a number** then would be one of the two equal factors of that number. The positive square root of a number is called the principal square root. The symbol used to indicate we are to find the square root of a number is \( \sqrt{\cdot} \) and is called a **radical sign**; the number under the radical sign in called the **radicand**.

An irrational number can not be written as a ratio. Examples of irrational numbers are:
The square root of any non-perfect square is an irrational number.
The value of \( \pi \) is an irrational number
A non-repeating, non-terminating decimal is an irrational number.

We can approximate values for irrational numbers with rounding. We sometimes say the value for \( \pi \) is \( 22/7 \) or 3.1416

When we want to find the approximate value of a non perfect square we use our perfect squares to help us.

If we want to find the square root of 17 we find two consecutive integers that the square root is between. So with the square root of 17. We know the square root of 16 and the square root of 25, the square root of 17 lies between 4 and 5, but closer to 4.
Chapter 1.2 The Set of Real Numbers

Real Numbers are the set of all rational and irrational Numbers

Natural numbers – counting numbers: 1, 2, 3, 4......
Whole numbers – counting numbers and 0: 0, 1, 2, 3, 4, 5......
Integers – Whole numbers and their opposites: ……-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5......
Rational numbers – integers, and all numbers that can be written as a/b where b≠0.

Completeness Property for Points on a Number Line

– Every Real number corresponds to exactly one point on the number line
– Every point on the number line corresponds to exactly one real number

The origin on the number line is 0, to the right of the origin are the positive numbers, and to the left of the origin are the negative numbers.

Not all numbers to the left and right of 0 on the number line are integers, -1/2 is not an integer. We call these numbers signed numbers.

Zero is neither a positive nor a negative number.

The absolute value of a number is the distance the number is from 0 on a number line.

The additive inverses are numbers that are on opposite sides of zero on the number line and the same distance away from zero on the number line. They are also called opposites. Opposites or additive inverses have the same absolute value. Not every number has an additive inverse, 0 does not have an inverse.

We can apply operations to numbers. Operations are addition, subtraction, multiplication and division. When we apply an operation to any number in a set and the result is also in the same set of numbers that set is said to be closed under that operation. For example, suppose we use the set of whole numbers and we ask the question “Is the set of whole numbers closed under addition?” Try some examples – give me two whole numbers and add them – do you get a whole number as the sum of the operation? Will you always get a whole number? The answer is yes – so the set of whole numbers is closed under the operation of addition.

To show that a set is not closed under a particular operation you just have to show one counterexample, or situation for which the result would not fall in the set. Take for example the set of natural numbers. And the operation subtraction. Is the set of natural numbers closed for subtraction? NO: 1-2 = -1 and -1 is not a natural number. One counterexample proves the statement is false.
Chapter 1.3 Add and Subtract Real Numbers

The rules for adding and subtracting signed numbers are the same as adding and subtracting integers. If you do not remember those rules you can use a number line to assist you.

**Adding signed numbers:**

- **like signs:**
  - Add their absolute value
  - use the sign of the addend for the sum

- **Unlike signs:**
  - subtract the lesser absolute value from the greater absolute value
  - Use the sign of the addend with the greater absolute value for the sum.

**Subtracting signed numbers:**

- Change the sign to addition and add the opposite

When adding and subtracting it is always a good idea to estimate your answer before you start – that allows you to check the reasonableness of your answer. If your estimate is too far from your actual that tells you that you need to go back and check your work – both your estimate and your sum or difference.

Chapter 1.4 Multiply and Divide signed numbers

Again like with addition and subtraction – the rules for multiplying and dividing signed numbers are the same as the rules for multiplying and dividing integers.

**Rules for multiplying signed numbers:**

- If the factors have like signs the product is positive
- If the factors have unlike signs the product is negative

**Rules for dividing signed numbers:**

- If the dividend and the divisor have like signs the quotient is positive
- If the dividend and the divisor have unlike signs the quotient is negative

**HINT – WRITE THE SIGN FOR YOUR PRODUCT OR QUOTIENT BEFORE YOU SOLVE**
Chapter 1.5 - 1.5A Properties of Integer Exponents

Exponents represent repeated multiplication. It is very important to remember that $2^6$ means $2 \times 2 \times 2 \times 2 \times 2 \times 2$ and not $2 \times 6$ (that is the most common mistake with exponents). The large number in the expression $2^6$ is called the base and the smaller superscript number is called the exponent. You multiply the base by itself the number of times indicated by the exponent.

We have some rules for exponents:

**Definition of Zero Exponent:** For any real number $a$, $a \neq 0$, $a^0 = 1$.
Any non-zero real number to the 0 power is 1.

**Example:**

$4^0 = 1$
$X^0 = 1$

**Law of Negative Exponent:** For any real number $a$, $a \neq 0$, $a^{-m} = \frac{1}{a^m}$.
Any non-zero number to the negative power is written as $1/that number to a positive power$. A negative exponent means the multiplicative inverse or reciprocal of the expression. Any time you flip a number with an exponent over the division line the power of that exponent changes sign.

**Example:**

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$(-2\times6)^{-2} = (-12)^{-2} = \frac{1}{(-12)^2} = \frac{1}{144}$

**Law of Exponents for Multiplication:** For any real number $a$, $a \neq 0$, and integers m, and n, $a^m \cdot a^n = a^{m+n}$.
That means that for any real numbers with the same base when you multiply your leave the base and adding the exponents.

**Example:**

$2^3 \cdot 2^4 = 2^{3+4} = 2^7$
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

**Law of Exponents for Division:** For any real number $a$, $a \neq 0$, and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$
That means that for any real numbers with the same base you divide by leaving the base the same and subtracting the exponents.

**Power of a Power Property:** $(a^m)^n = a^{m \cdot n}$ Any time you are raising a number to a power to another power you multiply the exponents.

**Examples:**

$(3^2)^2 = (3 \times 3)^2 = 9^2 = 81$ This is the same as $3^{2\cdot2} = 3^4 = 3 \times 3 \times 3 \times 3 = 81$

$(3^4)^2 = 3^{4 \cdot 2} = 3^8 = \frac{1}{3^8}$
**Power of a Product Property:** \((ab)^m = a^m \cdot b^m\) Any time you are raising the product of two or more numbers to a power you can raise each factor to the power and then multiply the results together.

Examples:

\[(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36\]
\[(3x)^3 = 3^3 \cdot x^3 = 27x^3\]

**Power of a Quotient Property:** \((\frac{a}{b})^m = a^m / b^m\), \(b \neq 0\). When a division expression is being raised to a power you can raise the dividend to the power, the divisor to the power and then divided the resulting two numbers.

Examples:

\[\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{2^2 \cdot 2^2}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{16}{81}\]
\[\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{3 \cdot 3 \cdot 3}{5 \cdot 5 \cdot 5} = \frac{27}{125}\]
Chapter 1.6 The Order of Operations

When a numerical expression involves two or more operations, there is a specific order in which these operations must be performed.

<table>
<thead>
<tr>
<th>When evaluating an expression, proceed in this order:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. parentheses are done first</td>
</tr>
<tr>
<td>2. exponents are done next</td>
</tr>
<tr>
<td>3. multiplication and division are done as they are</td>
</tr>
<tr>
<td>encountered from left to right.</td>
</tr>
<tr>
<td>4. addition and subtraction are done as they are</td>
</tr>
<tr>
<td>encountered from left to right.</td>
</tr>
</tbody>
</table>

The proper application of "order of operations" is needed when working with such mathematical topics as evaluating formulas, solving equations, evaluating algebraic expressions, and simplifying monomials and polynomials.

There is a phrase that may help you to remember this order: PEMDAS

Parenthesis, Exponents, (Multiplication/Division),
(Add/Subtract)

Please Excuse (My Dear) (Aunt Sally).

The reason (multiplication & division - MD) and (add & subtract - AS) are "grouped" in sets of parentheses is that when those operations are next to each other you do the math from left to right. You do not always do multiplication or addition first. It may be the case where division will be done BEFORE multiplication or subtraction will be done BEFORE addition.

Be very careful when listening to Aunt Sally!!!

If you forget to take MD and AS in order as you come to them from left to right, Aunt Sally's advice is toast!
Chapter 1.7 - Scientific Notation

Very large numbers or very small numbers can be represented using scientific notation.

To write a number in scientific notation you must:
1. Express one factor as a number greater than or equal to 1 but less than 10. Move the decimal point to the left or right.
2. Express the other factor as a power of 10. Count the number of places the decimal point was moved to the left or right and use the number as the exponent of the power of 10. If you move the decimal to the left the exponent will be positive and if you move the decimal to the right the exponent will be negative.

A number is in what is called standard form when you just list the digits that make up the number. Example: 298,000

A number is in scientific notation when you have it written as a product of two factors one being greater than 1 and less than ten and the other being a power of 10.

Example: 298,000 in scientific notation is 2.98 x 10^5.

To add or subtract using scientific notation:
- Add or subtract the decimal factors
- Multiply the sum or difference by the common power of 10.
- Make sure to write the sum in scientific notation. In order to do this you must remember your rules for exponents.

Example: (4.215 x 10^{-2}) + (3.2 x 10^{-4})
= (4.215 x 10^{-2}) + (0.032 x 10^{-2})
= (4.215 + 0.032) x 10^{-2}
= 4.247 x 10^{-2}

To Multiply or divide using scientific notation:
- Multiply or divide the decimal factors
- Multiply or divide the powers of 10.
- Simplify and write the product in scientific notation.

Example: (3.4 x 10^6)(4.2 x 10^3)
= (3.4)(4.2) x 10^{6+3}
= 14.28 x 10^9
= 1.4 x 10^{10}

Example: (6.4 x 10^6)/(8.9 x 10^2)
= (6.4)/(8.9) x 10^{6-2}
= 0.719 x 10^4
= 7.2 x 10^3
Chapter 1.8-1.8A Algebraic Expressions and Interpreting parts of expressions

A term in an algebraic expression is a number, a variable, or the product or quotient of a number and a variable. In the example $7.25h + 75$ we have two terms, $7.25h$ and $75$.

The numerical part of a term containing a number and a variable is called the **numerical coefficient** of the term. In our example $7.25$ is the **numerical coefficient** of the term $7.25h$.

The variable or variables of a term are called the **literal coefficients**. In our example $h$ is the literal coefficient.

A term that does not have a variable is called a **constant**. In our example $75$ is the constant.

**Certain words or phrases imply certain operations:**

<table>
<thead>
<tr>
<th>English</th>
<th>Mathlish</th>
</tr>
</thead>
<tbody>
<tr>
<td>is/was/will be&gt;equals/the result is</td>
<td>=</td>
</tr>
<tr>
<td>Quantity</td>
<td>( )</td>
</tr>
<tr>
<td>a number/what number/what fraction</td>
<td>n</td>
</tr>
<tr>
<td>the opposite of a number</td>
<td>- n</td>
</tr>
<tr>
<td>three consecutive numbers</td>
<td>n, $n+1$, $n+2$</td>
</tr>
<tr>
<td>sum/more than/increased by/and</td>
<td>+</td>
</tr>
<tr>
<td>difference/less than/decreased by</td>
<td>-</td>
</tr>
<tr>
<td>product/times/multiplied by/of</td>
<td>$\times$ or $\cdot$</td>
</tr>
<tr>
<td>quotient/over/divided by</td>
<td>$\div$ or fraction symbol “/”</td>
</tr>
<tr>
<td>what percent</td>
<td>$n/100$</td>
</tr>
</tbody>
</table>

When you are reading a math problem you need to read the problem multiple times. You need to stop after each sentence and take the time to understand the information you are given and what the problem is asking you to do. Translating the English language into math language requires patience and understanding, you need to put yourself in the problem situation and determine what you know and what you would do to solve the problem. Only with understanding can you translate an English expression into a mathematical expression.

If we want to evaluate an algebraic expression we just need to substitute in a numerical value for the variable and calculate. When we do this we are using the **substitution principal**. When you evaluate an algebraic expression you must remember to follow your rules for order of operations.
Chapter 1.9 – 1.9A Properties of Real Numbers and the sums and products of rational and irrational numbers.

**Closure:** for any two numbers $a + b$ is a unique number and $a \cdot b$ is a unique number (one point on the number line)

**Commutative:** $a + b = b + a$, $a \cdot b = b \cdot a$

It is like a commute to and from school. You drive from home to school and from school to home, both are the same distance. No matter what order you add two real numbers in the sum will be the same and no matter what order you multiply two real numbers the product will be the same.

**Identity property:** $a + 0 = a$ and $a \cdot 1 = a$.

If you add 0 to any number you get the same or identical number, if you multiply any real number by 1 you get the same or identical number.

**Inverse property:** For every real number $a$, there is a unique real number $-a$ such that $a + (-a) = 0$. For any real number $a$, there is a unique real number $1/a$ such that $a \cdot 1/a = 1$.

**Associative property:** $a + b + c = (a+b)+c = a+(b+c)$ or $a \cdot b \cdot c = (a \cdot b) \cdot c = a \cdot (b \cdot c)$.

I think of it like a car pool. When I pick up my neighbor I have Noah and Madeline in the car and I pick up Dan after. When Dan’s mom drives she has Dan in the car then she picks up Noah and Madeline. With the associative property the order the numbers are in does not change just the grouping does. With the commutative property the order of the numbers changes, however the grouping of the numbers does not.

**Distributive property:** $a \cdot (b+c) = a \cdot b + a \cdot c$ and $(b+c) \cdot a = b \cdot a + b \cdot c$

It is like when I distribute papers, I give one to each of you, but I always come back to my main pile to hand a sheet to the next person.

All of these properties are used when we simplify algebraic expressions. We sometimes have to combine like terms when working with algebraic expressions.

What does it mean to combine like terms? Like terms must have exactly the same variables with exactly the same exponents, but the numerical coefficients can be different.

**Definition of Subtraction** : $a - b = a + (-b)$ Subtracting is the same as adding the opposite.

Example: $4x+6y+3x-2y+8$

4x+6y+3x+(-2y)+8  definition of subtraction
4x+3x+6y+(-2y)+8  commutative property of addition
(4x+3x) + (6y+(-2y))+8  associative property to group like terms
(4+3)x+(6+(-2))y+8  distributive property
7x+4y+8  add coefficients of like terms.

The sum, difference or product of two integers is an integer.
The quotient of two integers is rational if the divisor is not equal to zero.
The sum or difference of an integer and an irrational number is irrational.
The product or quotient of a nonzero integer and an irrational number is irrational.